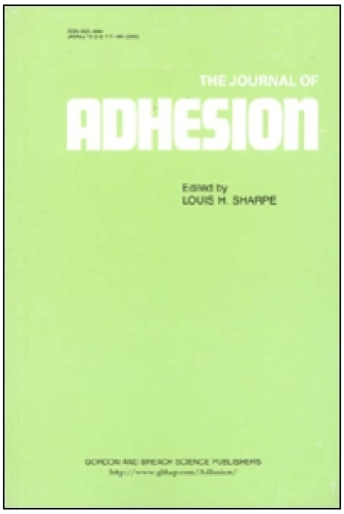

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On the Theory of the Stress—Strain State in Adhesive Joints

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The stress–strain state in adhesive joints is analyzed for normal stressing and an exact solution is obtained for the elastic-theory problem in these three-layer systems. The differential equations of equilibrium and boundary conditions are satisfied exactly or (on the surface where the stresses are given) integrally.

The solution indicates that in addition to the tensile stresses codirectional with the external force field, stresses perpendicular to the latter (i.e. codirectional with the joint itself) are also produced: compressive in the adherend, tensile in the adhesive. In addition, tangential stresses are produced in the end zone of each layer, rendering this zone the most dangerous.

The classical strength theory shows that the worst danger spot is the bulk of the adhesive rather than the interface. Failure criteria are obtained for the system in question, as well as conditions of strength equivalence of its components. The latter is effected by reducing the thickness of the adhesive layer as well as by regulating its physico-chemical characteristics

INTRODUCTION

In the general case, an adhesive joint represents a system of two solid bodies held together by forces acting by intermediary of the adhesive layer. Failure of the joint may occur either through detachment of the adhesive layer in intact form (adhesive failure) or through rupture of the layer or of one of the adherends (cohesive failure). The mechanism is identical in both cases, namely gradual fluctuative thermal destruction of cohesive or adhesive bonds—in fact, it does not basically differ from that of ordinary failure of solids.[1] In most cases of practical importance, cohesive failure is quasi-brittle, i.e. such that plastic deformation, although present, is confined to micro-regions adjoining the crack tip.[2–4] This is wholly the case for solid polymer adhesives, as is seen both from the absence of measurable plastic deformations[5,6] and from the morphology of the rupture surface[7] under conditions promoting failure of adhesive joints. Moreover, a thin layer of polymer adhesive enclosed between two solid bodies is capable of quasi-brittle failure even under time-temperature conditions at which the bulk polymer still exhibits plastic behavior. Accordingly, the quasi-brittle category covers the great majority of practical cases of failure in adhesive joints. In these circumstances, the loading and failure processes may be treated by means of the elastic theory and the generalized crack theory. Obviously, it would be desirable to take the plastic deformation into account[8,9] but no quantitative tool is as yet available for this purpose. However, in our case this is not essential, insofar as we deal with the energy aspect of the joint, irrespective of the specific failure mechanism.

1. DETERMINATION OF THE STRESS–STRAIN STATE IN ADHESIVE JOINTS BY MEANS OF THE ELASTIC THEORY

Consider a system consisting of a pair of sheets (1 and 3) of solid materials (adherends) rigidly jointed through a layer (2) of solid adhesive. The adherends are assumed to have identical elastic properties.

Notation

(Figure 1) (Indices 0 and 1 refer to adhesive and adherend respectively).
E, ν—elastic modulus and Poisson's ratio, respectively.
$2h$—thickness of adhesive layer.
$2H$, $2b$, $2l$—thickness, width and length of joint, respectively.

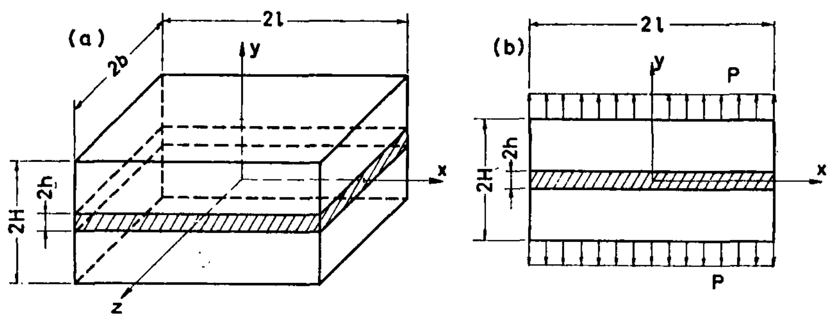

FIGURE 1 Adhesive joint under uniform load (rupture mode). (Schematic).

The system is subjected to a uniformly-distributed tensile load per unit surface, denoted P, applied at the boundaries $y = \pm H$. Obviously, the stress–strain state and consequently the limit load parameter P depend on the elastic properties of all components.

According to the elastic theory, the set of differential equations of equilibrium for each layer, in terms of displacement components, reads[10,11]

$$
\begin{aligned}
(\lambda_j+\mu_j)\frac{\partial\theta_j}{\partial x}+\mu\Delta u_j &= 0\\
(\lambda_j+\mu_j)\frac{\partial\theta_j}{\partial y}+\mu\Delta v_j &= 0\\
(\lambda_j+\mu_j)\frac{\partial\theta_j}{\partial z}+\mu\Delta w_j &= 0 \quad (j = 0.1)
\end{aligned}
\tag{1.1}
$$

where u_j, v_j, w_j are the components of the displacement vector, θ_j—the first invariant of the strain tensor, λ_j, μ_j—Lamé's constants.

The solution is sought in the following form:

$$u^{(0)} = u^{(1)} = w^{(0)} = w^{(1)} = \frac{P}{E_0}Ax \tag{1.2}$$

$$v^{(0)} = \frac{P}{E_0}By; \qquad v^{(1)} = \frac{P}{E_0}(Cy+Dh). \tag{1.3}$$

Here $u^{(1)} = u^{(0)}$, as all points in planes parallel to those of the coordinates before deformation remain in them after it, i.e. all points in a given plane have the same displacement irrespective of where they belong. This equality of displacement is, in fact, the cause of the stresses to be discussed later.

The factor $1/E_0$ was introduced for mathematical convenience.

Since for $y = h$, $v^{(0)} = v^{(1)}$, we have

$$B = C+D$$

$$\varepsilon_x^{(0)} = \varepsilon_x^{(1)} = \varepsilon_z^{(0)} = \varepsilon_z^{(1)} = \frac{P}{E_0}A; \qquad \varepsilon_y^{(0)} = \frac{P}{E_0}B; \qquad \varepsilon_y^{(1)} = \frac{P}{E_0}C \tag{1.4}$$

and for the stress components

$$
\begin{aligned}
\sigma_{xx}^{(0)} = \sigma_{zz}^{(0)} &= 2\mu_0\varepsilon_x^{(0)}+\lambda_0\theta_0 = \frac{P}{(1+\nu_0)(1-2\nu_0)}(A+\nu_0 B)\\
\sigma_{xx}^{(1)} = \sigma_{yy}^{(1)} &= 2\mu_1\varepsilon_x^{(1)}+\lambda_1\theta^{(1)} = \frac{mP}{(1+\nu_0)(1-2\nu_1)}(A+\nu_1 C) \qquad m = \frac{E_1}{E_0};
\end{aligned}
\tag{1.5}
$$

$$\sigma_{yy}^{(0)} = \sigma_{yy}^{(1)} = P \tag{1.6}$$

Equation (1.6) follows from the equilibrium condition for any part of the joint sectioned perpendicular to the y-axis.

In order to determine the constants A, B and C, we express the components $\sigma_{yy}^{(0)}$ and $\sigma_{yy}^{(1)}$ through them:

$$\sigma_{yy}^{(0)} = \frac{P}{E_0}[\lambda_0(A+B)+2\mu_0 B] = P \tag{1.7}$$

$$\sigma_{yy}^{(1)} = \frac{P}{E_0}[\lambda_1(A+C)+2\mu_1 C] = P \tag{1.8}$$

At the edges $x = \pm l$, boundary conditions can be satisfied only integrally, i.e. we stipulate that the normal stresses σ_{xx}, summed, do not produce a longitudinal force. This is the third boundary condition necessary for determining the constants. It yields

$$\alpha\sigma_{xx}^{(0)}+(1-\alpha)\sigma_{xx}^{(1)} = 0 \qquad (1.9)^{\dagger}$$

where

$$\alpha = \frac{h}{H} \qquad (1.10)$$

The boundary conditions (1.6) and (1.9) in turn yield

$$A = -\frac{\alpha(1-\nu_1)\nu_0+\nu_1(1-\alpha)(1-\nu_0)}{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)} \qquad (1.11)$$

$$B = \frac{\alpha(1-\nu_1)+(1-\alpha)[m(1+\nu_0)(1-2\nu_0)+2\nu_0\nu_1]}{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)} \qquad (1.12)$$

$$C = \frac{\alpha(1-\nu_1-2\nu_1^2+2m\nu_0\nu_1)+m(1-\alpha)(1-\nu_0)}{m[\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)]} \qquad (1.13)$$

and finally we have:

$$\sigma_{xx}^{(0)} = \sigma_{zz}^{(0)} = \frac{P(1-\alpha)(m\nu_0-\nu_1)}{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)} \qquad (1.14)$$

$$\sigma_{xx}^{(1)} = \sigma_{zz}^{(1)} = \frac{-P\alpha(m\nu_0-\nu_1)}{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)} \qquad (1.15)$$

$$\sigma_{yy}^{(0)} = \sigma_{yy}^{(1)} = P$$

The above equations show that for $m > 1$ and $\nu_0 \geqslant \nu_1$, we have $\sigma_{xx}^{(0)} > 0$, i.e. the adhesive undergoes tension, and the adherend—compression.

The stresses $\sigma_{xx}^{(0)}$ and $\sigma_{xx}^{(1)}$ are independent of x; in these circumstances the solution is exact only if at $x = \pm l$ the normal stresses are:

$$N_x^{(0)} = \sigma_x^{(0)}, \qquad N_x^{(1)} = \sigma_x^{(1)}$$

These forces have a zero resultant and zero moment. Consequently (by St. Venant's principle) their effect on the stresses over a certain distance from $x = \pm l$ is negligible. (These stresses decrease as the edges are approached).

For the equilibrium of an element close to the edges, tangential stresses must be applied, increasing as the edges are approached. These stresses promote failure in the edge zone and render it the most dangerous potentially.

† By St. Venant's principle, the solution is exact at sufficient distance from the edges. (Details will be discussed later.) Elsewhere, at any point, the equilibrium equations are completely satisfied.

The above analysis shows that apart from the tensile stresses codirectional with the external load field, the joint undergoes both tensile (in the adhesive) and compressive (in the adherend) stresses perpendicular to that field, i.e. codirectional with the layer. This conclusion confirms that obtained in Ref. [12] for a cylindrical joint. In addition, tangential stresses set in near the edge of each layer.

2. DETERMINATION OF THE LIMIT LOAD BY MEANS OF THE CLASSICAL STRENGTH THEORIES FOR BRITTLE MATERIALS

Given the stress state at a certain point in a brittle or "moderately plastic" material, the equivalent stress responsible for failure is[13]

$$\sigma_{eq} = \sigma_1 - f\sigma_3 = \sigma_1\left(1 - f\frac{\sigma_3}{\sigma_1}\right) \tag{2.1}$$

where σ_1 and σ_3 are the major and minor principal stresses respectively. By the classical hypothesis of maximum uniaxial elongation $f = \nu(1+\sigma_2/\sigma_3)$, or, as is more commonly accepted today, equals the ratio of the ultimate tensile and compressive strengths:

$$f = \sigma_{bt}/\sigma_{bc} \tag{2.2}$$

Substituting $\sigma_{yy}^{(0)}$, $\sigma_{yy}^{(1)}$, $\sigma_{xx}^{(0)}$ and $\sigma_{xx}^{(1)}$ from (1.14) and (1.15) in the expressions

$$\sigma_{eq}^{(0)} = \sigma_{yy}^{(0)} - f_0\sigma_{xx}^{(0)} \quad \text{for the adhesive}$$

and

$$\sigma_{eq}^{(1)} = \sigma_{yy}^{(1)} - f_0\sigma_{xx}^{(1)} \quad \text{for the adherend,}$$

we find

$$\sigma_{eq}^{(0)} = \frac{P\{\alpha(1-\nu_1)+(1-\alpha)[m(1-\nu_0-f_0\nu_0)+f_0\nu_1]\}}{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)} \tag{2.3}$$

$$\sigma_{eq}^{(1)} = \frac{P[m(1-\alpha)(1-\nu_0)+\alpha(1-\nu_1-f_1\nu_1+mf_1\nu_0)]}{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)} \tag{2.4}$$

It is seen that $\sigma_{eq}^{(0)}$ increases with α and the limit load P decreases. This conclusion is fully confirmed by experimental data on the effect of adhesive thickness on the strength of the joints.

At $m \gg 1$ and $\alpha \to 0$,

$$\sigma_{eq}^{(0)} = P\left[1 - \frac{\nu_0}{1-\nu_0}f_0\right] \tag{2.5}$$

i.e., the equivalent stress decreases with increasing f_0 and the limit load increases.†

Analytically for the adherend, at $m \gg 1$ and $\alpha \to 0$

$$\sigma_{eq}^{(1)} = \left[1 + \frac{\alpha f_1 \nu_0}{(1-\alpha)(1-\nu_0)}\right] P \tag{2.6}$$

i.e., the equivalent stress increases with increasing f_1 and α, and the limit load decreases; increase of ν_0 has the reverse effect.†

The limit load for the adhesive is given by

$$\sigma_{eq}^{(0)} = \sigma_{bt}^{(0)} \tag{2.7}$$

where $\sigma_{bt}^{(0)}$ is as defined earlier (p. 69). Substituting $\sigma_{eq}^{(0)}$ from Eq. (2.3), we have

$$P^{(0)} = \frac{[\alpha(1-\nu_1) + m(1-\alpha)(1-\nu_0)]\sigma_{bt}^{(0)}}{\alpha(1-\nu_1) + (1-\alpha)[m(1-\nu_0 - f_0\nu_0) + f_0\nu_1]} \tag{2.8}$$

Analogically, for the adherend

$$\sigma_{eq}^{(1)} = \sigma_{b,c}^{(1)}$$

and

$$p^{(0)} = \frac{[\alpha(1-\nu_1) + m(1-\alpha)(1-\nu_0)]\sigma_{bt}^{(0)}}{m(1-\alpha)(1-\nu_0) + \alpha(1-\nu_1 - f_1\nu_1 + m f_1 \nu_0)} \tag{2.9}$$

The ratio of the limit loads is thus:

$$\kappa = \frac{m(1-\alpha)(1-\nu_1) + \alpha(1-\nu_1 - f_1\nu_1 + m f_1\nu_0)}{\bar{\sigma}[\alpha(1-\nu_1) + (1-\alpha)[m(1-\nu_0 - f_0\nu_0) + f_0\nu_1]]}; \qquad \bar{\sigma} = \frac{\sigma_{bt}^{(1)}}{\sigma_{bt}^{(0)}} \tag{2.10}$$

The parameter κ is one of the failure criteria for an adhesive joint. At $\kappa > 1$ failure occurs in the adherend, at $\kappa < 1$—in the adhesive. At $\kappa = 1$ the system is of uniform strength, and solving (2.10) for α, we obtain

$$\alpha = \frac{\bar{\sigma}[m(1-\nu_0 - f_0\nu_0) + f_0\nu_1] - m(1-\nu_1)}{1 - \nu_1 - f_1\nu_1 + m[\nu_0(1+f_1) - 1] + \bar{\sigma}[m(1-\nu_0 - f_0\nu_0) + f_0\nu_1 + \nu_1 - 1]} \tag{2.11}$$

If the elastic constants of the materials are such that $0 < \alpha < 1$, a joint of uniform strength is obtainable, the correct thickness of the adhesive layer being

$$2h_0 = 2\alpha H \tag{2.12}$$

Seeing that in most practical cases the adhesive is the weakest component; the conclusion from the above is that, for improved strength of the joint, the material chosen as adhesive should have maximum $\sigma_{b,t}/\sigma_{b,c}$ and ν_0—in

† No confirmatory experimental data available to the authors.

addition to the familiar solution of reducing its thickness. Figure 2 shows the dependence of the equivalent stresses in the adhesive layer relative to the lead P. These stresses vary with the parameter $m(m = E_1/E_0)$, $\nu_0 = 0.47$, $\nu_1 = 0.3$ for all curves. A family of curves is shown for $f_0 = 0.5$ and different values of the parameter α (0.05, 0.1, 0.15, 0.2, 0.25, 0.3), and a similar group is shown for $f_0 = 1$.

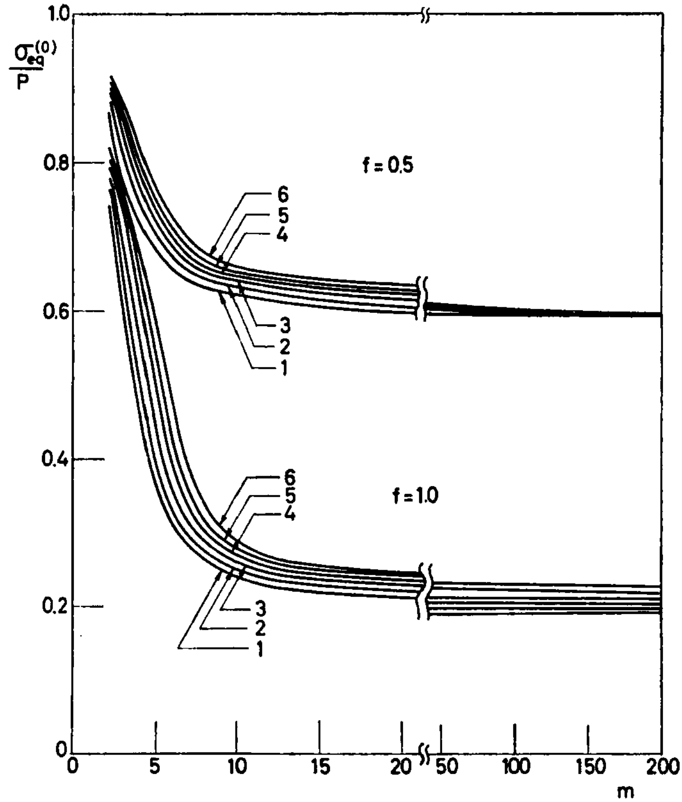

FIGURE 2 Equivalent stress vs. ratio M, $f = 0.5$, 1.0, for different values of α (1–0.5; 2–0.1; 3–0.15; 4–0.2; 5–2.5; 6–0.3).

The stresses in the adhesive layer are expressed in terms of $\sigma_{eq}^{(0)}/P$ where $\sigma_{eq}^{(0)}$ is the equivalent stress in the adhesive layer and P the stress that would obtain in such a material by itself. This ratio is seen to decrease with increase of m and f, as well as with decrease of α.

Decrease of the equivalent stress in the adhesive joint is due to tensile stresses in the plane perpendicular to the field of external loads.

This effect becomes more marked in adhesive materials of increasing plasticity. The stress analysis of the adhesive joint in terms of α confirms the advisability of reducing the thickness of the adhesive layer and thereby the ratio $\sigma_{eq}^{(0)}/P$. Thus the given data leads to the following recommendations for adhesive joints in materials whose failure under uniaxial tension is of brittle nature.

The adhesive should be made from plastic materials with maximal Poisson's ratio and large m.

The thickness of the adhesive layer should be small. Failure of the adhesive joint is most likely along the adhesive layer, and the limit load may be much higher than that measured on specimens of the adhesive material by itself.

3. DETERMINATION OF CRITICAL STRESSES IN ADHESIVE JOINTS BY MEANS OF THE GENERALIZED CRACK THEORY

According to the crack theory, failure is due to transformation of a nucleus into a major crack, followed by growth of the latter. Mathematically, the crack is considered as a ruptured surface of displacements in the deformed body.†

In the linear elastic theory, the stress state in a cracked body under a given system of external load is represented by the sum of solutions of two statical problems. The first problem refers to an intact (uncracked) body under external load, excluding the forces acting on the lips of the crack; the second—to the same body in its cracked state, under the latter forces only. These consist of the external load on the lips, plus forces equal and opposite to the stresses produced along the crack under the conditions of the first problem.

In the case of a plane surface with a rectilinear crack (parallel to the x-axis) free from external load, the stress and displacement components are obtainable from asymptotic formulae. Near the tips of the crack, the stress components depend exclusively on the intensity coefficients k_1 and k_2, given[4] by:

$$
\begin{aligned}
k_1 &= \frac{1}{(\pi c)^{\frac{1}{2}}} \int_{-c}^{c} \sigma_{yy}(x, 0)\left(\frac{c+x}{c-x}\right)^{\frac{1}{2}} dx \\
k_2 &= \frac{1}{(\pi c)^{\frac{1}{2}}} \int_{-c}^{c} \sigma_{xy}(x, 0)\left(\frac{c+x}{c-x}\right)^{\frac{1}{2}} dx
\end{aligned}
\tag{3.1}
$$

where c is half the crack length, and $\sigma_{yy}(x, 0)$, $\sigma_{xy}(x, 0)$ are known finite functions, representing the distribution of normal and tangential stresses along the lips at $y = 0$ and obtainable from the elastic solution to the first problem.

† A current overview of mathematical methods in fracture theory is found in Refs. [14–16]. Discussion of the strength of adhesive joints from an approach of fracture theory can be found in Refs. [17, 18]. Problems dealing with the strength of structures composed of two thin, infinite elastic films adhered with a thick film of adhesive along the entire surface, except for a thin strip, have been investigated in Ref. [19].

Near the tips the lips of the crack converge, and surface forces of cohesion become prominent. These forces increase with the load, and at a certain critical level of the latter cause sudden escalating widening of the crack.

The problem of crack equilibrium is thus formulated as follows:

Given the system of initial defects (crack nuclei in the body) and the mode of loading, depending on a single monotonically increasing parameter. Find the critical level of this parameter, at which the cracks widen.

This critical level is given by Truin's formula

$$k_1^2+k_d^2 = \frac{E\gamma}{1-\nu^2} \tag{3.2}$$

where γ is the surface energy density characterizing the cohesive effect (the energy expended in forming a unit area of a new surface).

In the case of an elastic plane surface as above under uniform tension P, we have

$$k_1 = \frac{P}{(\pi c)^{\frac{1}{2}}}\int_{-c}^{c}\left(\frac{c+x}{c-x}\right)^{\frac{1}{2}} dx = P(\pi c)^{\frac{1}{2}} \tag{3.3}$$

As was shown earlier, an adhesive joint under uniaxial tension undergoes also normal stresses σ_{xx}-tensile in the adhesive, compressive in the adherend; the former of these reduce the stress concentration near the crack tips, the latter increase it. Accordingly, we have

$$k_1 = P(\pi c)^{\frac{1}{2}}\left(1-f^*\frac{\sigma_{xx}}{\sigma_{yy}}\right) \tag{3.4}$$

where the factor f^* depends on the elastic properties of the material (see Section 4 below).

Substituting (1.14) and (1.15) in (3.4), we find

$$k_1^{(0)} = (\pi c)^{\frac{1}{2}}P\frac{\alpha(1-\nu_1)+(1-\alpha)[m(1-\nu_0-f_0^*\nu_0)+f_0^*\nu_1]}{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)} \tag{3.5}$$

and

$$P_{cr}^{(0)} = \frac{\alpha(1-\nu_1)+m(1-\alpha)(1-\nu_0)}{\alpha(1-\nu_1)+(1-\alpha)[m(1-\nu_0-f_0^*\nu_0)+f_0^*\nu_1]}\left[\frac{E_0\gamma_0}{\pi c(1-\nu_0^2)}\right]^{\frac{1}{2}} \tag{3.6}$$

which at $m \gg 1$, $\alpha \to 0$ reduces to

$$P_{cr} = \frac{1}{1-\dfrac{f_0^*\nu_0}{1-\nu_0}}\left[\frac{E_0\gamma_0}{\pi c(1-\nu_0^2)}\right]^{\frac{1}{2}} \tag{3.7}$$

i.e. $P_{cr}^{(0)}$ decreases with increasing α, and increases with increasing ν_0 and f_0^*. Near the edges $x = \pm l$, σ_{xx} vanishes and $P_{cr}^{(0)}$ decreases, so that crack growth and failure set in there at a lower load level.†

† This conclusion is again in agreement with practical experience.

For the adherend, we find analogically

$$P_{cr}^{(1)} = \frac{m(1-\alpha)(1-\nu_0)+\alpha(1-\nu_1)}{m(1-\alpha)(1-\nu_0)+\alpha(1-\nu_1-\nu_1 f_1^* + m f_1^* \nu_0)}\left[\frac{E_1\gamma_1}{\pi c(1-\nu_1^2)}\right]^{\frac{1}{2}} \tag{3.8}$$

so that the critical load ratio is

$$\kappa = \frac{m(1-\alpha)(1-\nu_0)+\alpha(1-\nu_1-\nu_1 f_1^* + m f_1^* \nu_0)}{\alpha(1-\nu_1)+(1-\alpha)[m(1-\nu_0-f_1^*\nu_0)+f_1^*\nu_1]}\left[\frac{\gamma_0(1-\nu_1^2)}{m\gamma_1(1-\nu_0^2)}\right]^{\frac{1}{2}} \tag{3.9}$$

Obviously, uniform crack growth in both components is conditional on $\kappa = 1$. At $\kappa > 1$ failure occurs in the adherend, at $\kappa < 1$—in the adhesive.

Equating κ to unity, we obtain an equation in α, which yields

$$\alpha = \frac{m(1-\nu_0-f_0^*\nu_0)+f_0^*\nu_1-m\bar{\gamma}(1-\nu_0)}{\bar{\gamma}[1-\nu_1-\nu_1 f_1^*+m f_1^*\nu_0-m(1-\nu_0)]+ m(1-\nu_0-f_0^*\nu_0)+f_0^*\nu_1+\nu_1-1} \tag{3.10}$$

where

$$\bar{\gamma} = \left[\frac{\gamma_0(1-\nu_1^2)}{m\gamma_1(1-\nu_0^2)}\right]^{\frac{1}{2}} \tag{3.11}$$

If the elastic constants, as well as f_0^* and f_1^*, are such that α is positive and less than unity, the materials can be combined to form a three-layer system of uniform strength, the proper thickness of the adhesive layer again being $2h_0 = 2\alpha H$ [see (1.12) above].

In conclusion, the generalized crack theory enables us to find expressions for the stressing intensity coefficient and the critical load parameter; to confirm earlier conclusions regarding the favorable effect of reduced adhesive thickness and regarding reduction of the critical load level at $x = \pm l$; finally, to formulate a criterion for the mode of failure of the joint.

4. DETERMINATION OF THE CRITICAL STRESS BY MEANS OF THE COMBINED ELASTIC AND CRACK THEORIES

If the external load is not uniaxial, the elastic elements may formally be represented as two fields. The first field refers to the stresses produced in an intact body under internally applied loads; the second—to those produced in the cracked body under symmetric compensating loads normal to the crack edges and equal to σ_{eq}. The asymptotic equations for the stress components near the tips are again based on Irwin's formula, with $\sigma_{yy}^{(eq)}$ substituted for σ_{yy}:

$$k_1 = \int_{-c}^{c} \sigma_{eq}\left(\frac{c+\alpha}{c-x}\right)^{\frac{1}{2}} dx = (\sigma_{yy}-f^*\sigma_{xx})(\pi c)^{\frac{1}{2}} \tag{4.1}$$

For σ_{yy} = const. and σ_{xx} = const. [given by (1.14) and (1.15)], with $f_j^* = f_j$ ($j = 0, 1$); f is given by (2.2), or equals Poisson's ratio (see p. 7 above).

In conclusion, the combined approach enables us to determine the hitherto unknown factor f^* which in turn makes it possible to find k, P_{cr} and κ.

5. DETERMINATION OF THE CRITICAL LOADS AT $x = \pm l$

As already noted, Eqs. (3.6) and (3.8) indicate that σ_{xx} vanishes on approaching $x = \pm l$ and P_{cr} consequently decreases. As regards this effect in an adhesive joint, we consider elements of the system, taken, respectively, from the bulk of one of the components (Figure 3a) and symmetrically about the

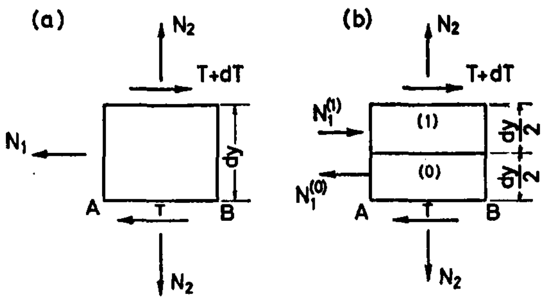

FIGURE 3 Edge element of adhesive joint: (a) in adhesive layer; (b) at interface.

interface (Figure 3b). The length of the elements is such that $\sigma_x^{(0)}$, $\sigma_x^{(1)}$ [as per (1.14)] remain constants, their width is unity, and their depth dy. In the first case, the following normal and tangential forces are involved (Figure 3a)

$$N_1 = \sigma_{xx}^{(0)}\,dy \cdot 1; \qquad N_2 = \sigma_{yy}^{(0)} AB \cdot 1 \tag{5.1}$$

$$T = -\sigma_{xx}^{(0)}\frac{dy}{2}; \qquad \frac{dT}{dy} = \sigma_{xx}^{(0)} \tag{5.2}$$

and in the second (Figure 3b)

$$N_1^{(0)} = \sigma_{xx}^{(0)}\frac{dy}{2}; \qquad N_1^{(1)} = |\sigma_{xx}^{(1)}|\frac{dy}{2}; \tag{5.3}$$

$$N_2^{(0)} = N_2^{(1)} = \sigma_{yy} AB \cdot 1 \tag{5.4}$$

$$\begin{aligned} T &= -[3\sigma_{xx}^{(0)} - |\sigma_{xx}^{(1)}|]\frac{dy}{8} \\ \frac{dT}{dy} &= \tfrac{1}{2}[\sigma_{xx}^{(0)} - |\sigma_{xx}^{(1)}|] \end{aligned} \tag{5.5}$$

It is seen that as the interface is approached from below, the tangential stresses and their y-gradient decrease. Thus, the bulk of the adhesive (specifically its middle), rather than the interface, is the worst danger spot.

In addition, Figure 2 shows that in this section the gradient of σ_{xx} reaches maximum, as established by other authors.[12]

CONCLUSIONS

The following conclusions may be drawn with regard to the stress-strain state of an adhesive joint under uniaxial tension:

1) The elastic theory shows that in addition to the tensile stresses codirectional with the external force field, stresses perpendicular to the latter are also produced: compressive in the adherent, tensile in the adhesive.

In addition, at $x = \pm l$, tangential stresses are produced, rendering this zone the most dangerous. Equilibrium analysis of edge elements taken, respectively, from the bulk of one of the components and symmetrically about the interface, yielded the normal and tangential stresses involved and showed that the bulk of the adhesive, rather than the interface, is the worst danger spot.

2) The classical strength theory shows that for improved strength of the joint, the material chosen as adhesive should have a maximum ultimate-strength ratio (tensile to compressive) and a maximum Poisson ratio—in addition to the familiar solution of reducing its thickness (which also follows from these considerations).

3) The generalized crack theory yields the stressing intensity coefficient (k) and the critical load parameter (P_{cr}); it also confirms the favorable effect of reduced adhesive thickness, indicates that the edge zone is the most dangerous, and provides a criterion for the mode of failure of the joint (κ). Used in conjunction with the classical theories, it permits determination of the hitherto unknown factor f in the formulae for k, P_{cr} and κ, thereby rendering them obtainable both analytically and experimentally.

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